Computer-Aided Engineering

Adapted from

C.N.Nightingle and J.K.Fidler ., Computer-Aided Circuit and System Design.,U.K +IEEE papers

Lecture 5

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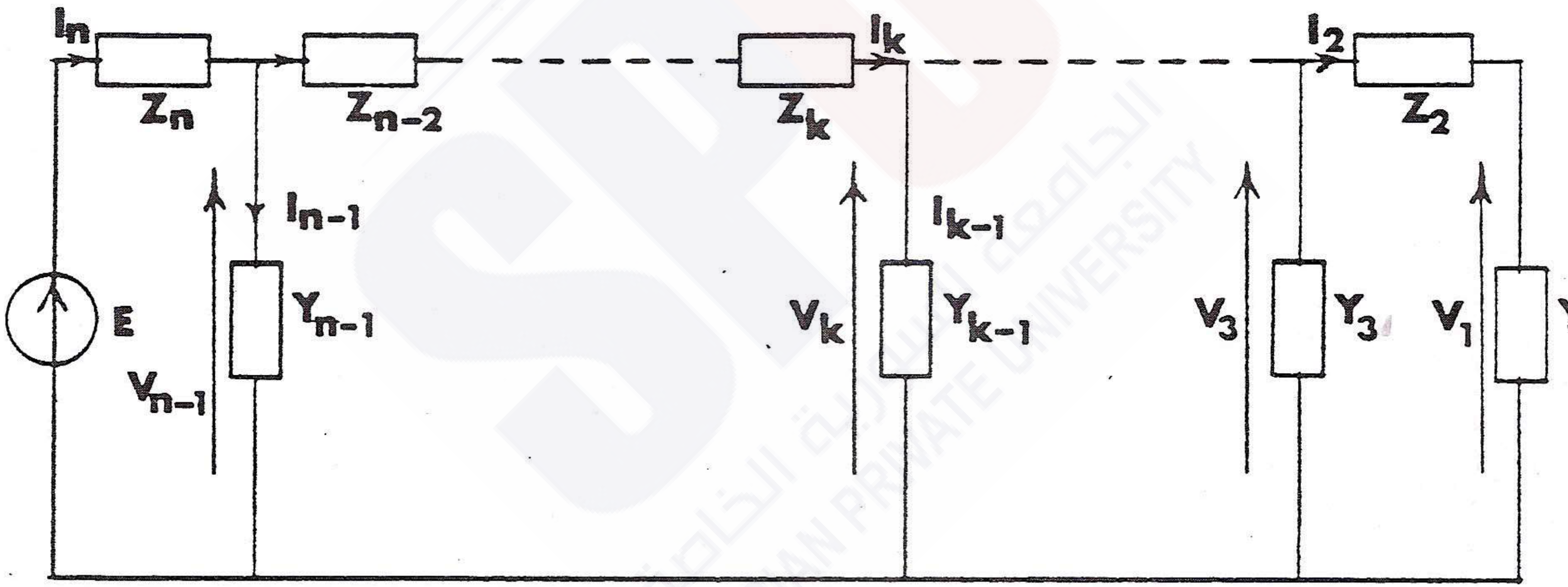
Bashkow's Analysis Method for Ladder Networks

Consider a ladder network consisting of n branches and excited by a voltage source E as shown in Figure



General n-branch doubly terminated ladder network

Bashkow's Analysis Method for Ladder Networks

Applying Kirchhoff's voltage and current laws alternately leads to the following equations:-

$$E = I_n Z_n + V_{n-1}$$

$$0 = -I_n + V_{n-1} Y_{n-1} + I_{n-2}$$

$$0 =$$

$$\cdot$$

$$\cdot \qquad (2.1)$$

$$\cdot$$

$$\cdot$$

$$0 = -V_3 + Z_2 I_2 + V_1$$

$$0 = -I_2 + V_1 Y_1$$

Bashkow's Analysis Method for Ladder Networks

these equations may be rewritten in matrix notations form as:-

$$
\begin{vmatrix} E \\ 0 \\ 0 \\ \\ 0 \\ \\ 0 \\ 0 \\ 0 \end{vmatrix}
=
\begin{vmatrix}
Z_n & 1 & 0 & \cdots\cdots & & & & \\
-1 & Y_{n-1} & 1 & \cdots\cdots & & & & \\
0 & -1 & Z_{n-2} & & & & & \\
 & & & Y_k & & & & \\
 & & & & \ddots & & & \\
 & & & & -1 & Y_3 & 1 & 0 \\
 & & & & 0 & -1 & Z_2 & 1 \\
 & & & & & 0 & -1 & Y_1
\end{vmatrix}
\begin{vmatrix} I_n \\ V_n{-1} \\ I_{n-2} \\ \\ V_k \\ \\ V_3 \\ I_2 \\ V_1 \end{vmatrix}
\qquad (2.2)
$$

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or simply as:- $b = A.x$ (2.3)

It can be shown that the determinant of matrix A can be written as a sequence of the following determinant equations:-

$$d_1 = Y_1$$

$$d_2 = Z_2 Y_1 + 1$$

$$d_3 = Y_3 (Z_2 Y_1 + 1) + Y_1 = Y_3 d_2 + d_1 \qquad (2.4)$$

.
.
.

$$d_n = (Z_n \text{ or } Y_n) \, d_{n-1} + d_{n-2}$$

assuming that $d_0 = 1$, and $d_{-1} = 0$. d_n is the value of the determinant of A. Now assuming Cramer's rule to solve equation (2.3) for the x_i, i.e. I_i or V_i, where $i = 1, 2, 3, \ldots, n$, gives:-

$$x_i = \frac{\Delta_i}{\Delta} \tag{2.5}$$

where $\Delta = d_n$, and Δ_i is the matrix in which the i'th column of matrix A is replaced by vector b.

From Figure 2.1, the voltage transfer function of the ladder is:-

$$F(s) = \frac{V_1}{E} \tag{2.6}$$

Using Cramer's rule to find the voltage transfer function leads to the following;-

1. Replace the b-matrix in column n of A as:-

$$
\Delta_n = \begin{vmatrix}
Z_n & 1 & 0 & 0 & \cdots & & 0 & E \\
-1 & Y_{n-1} & 1 & & & & & 0 \\
0 & -1 & Z_{n-2} & 1 & & & & 0 \\
 & & & & & & & \\
 & 0 & & & & & & \\
 & & & & & & & \\
 & & & & -1 & Y_3 & 1 & 0 \\
 & & & & & -1 & Z_2 & 0 \\
0 & 0 & & & & 0 & -1 & 0
\end{vmatrix} \qquad (2.7)
$$

or

$$\Delta_n = -E \begin{vmatrix} -1 & Y_{n-1} & 1 & 0 & \cdots\cdots & & 0 \\ 0 & -1 & Z_{n-2} & 1 & 0 & \cdots & 0 \\ & 0 & -1 & & & & \\ & & & & & & \\ & & & & & & Z_2 \end{vmatrix} \qquad (2.8)$$

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or

$$\Delta_n = -E \cdot M \qquad (2.9)$$

The determinant of matrix M is equal to -1 as the product of the principle diagonal elements, i.e.

$$\Delta_n = E \qquad (2.10)$$

therefore $F(s) = \frac{V_1}{E} = \frac{\Delta_n}{\Delta} \cdot \frac{1}{E} = \frac{1}{\Delta}$. Or in general

$$(V_k \text{ or } I_k)/E = d_{k-1}/\Delta = \frac{d_{k-1}}{d_n} \qquad (2.11)$$

Bashkow's Analysis Method for Ladder Networks

Each branch (Z_k or Y_k) IM_k will be considered as a ratio of two polynomials in the complex frequency variable, i.e.

$$IM_k = \frac{N_k(s)}{D_k(s)} \qquad (2.12)$$

Substituting (2.12) into individual sequence of the determinants d_i of (2.4) gives:-

Bashkow's Analysis Method for Ladder Networks

$$d_1 = \frac{N_1(s)}{D_1(s)}$$

$$d_2 = \frac{N_2(s)}{D_2(s)} \cdot \frac{N_1(s)}{D_1(s)} + 1$$

$$= \frac{N_2(s)N_1(s) + D_1(s)D_2(s)}{D_1(s)D_2(s)}$$

or in general

$$d_k = \frac{A_k(s)}{B_k(s)} = \frac{N_k(s)}{D_k(s)} \cdot \frac{A_{k-1}(s)}{B_{k-1}(s)} + \frac{A_{k-2}(s)}{B_{k-2}(s)} \qquad (2.13)$$

where $$A_k(s) = D_k(s) D_{k-1}(s) \ldots D_1(s)$$

hence $$A_{k-1}(s) = N_{k-1}(s) \; A_{k-2}(s) \tag{2.14}$$

and $$B_{k-1}(s) = D_{k-1}(s) \; B_{k-2}(s)$$

Taking into consideration equations (2.14) and substituting them into equation (2.13), it follows that an expression is obtained without common factor as:-

$$d_k = \frac{N_k(s) \; A_{k-1}(s) + D_k(s) \; D_{k-1}(s) \; A_{k-2}(s)}{D_k(s) \; D_{k-1}(s)} \tag{2.15}$$

where $A_O = B_O = D_O = 1$ and $A_{-1} = 0$, $k = 1, 2, \ldots, n$.

Thus:-

$$F(s) = \frac{V_1}{E} = \frac{1}{d_n} = \frac{B_n(s)}{A_n(s)}$$

$$= \frac{D_n(s) \; D_{n-1}(s) \; \ldots \; D_1(s)}{A_n(s)} \tag{2.16}$$

The formulation of (2.15) provides a simply programmed method of evaluation $d_k = \frac{A_k(s)}{B_k(s)}$, therefore the required voltage transfer function is found as:-

$$F(s) = \frac{B_k(s)}{A_k(s)} \qquad (2.17)$$

Equation (2.11) can be used to evaluate the other network functions:

in the present application, only the voltage transfer function is required.

```
Input data corresponding LCR-prototype topology
*************************************************
No.branches?4
Analyses required till termination ?1E12
No.coeffs in num & den (Max:14,14) ?3,4
Num coeffs(specification):(Lowest degree 1st):
1,0.112525,0.123951
Den coeffs(specification):(lowest degree 1st)=1:
1,1.463014,1.091287,0.574221
Req'd den const(DCFL):?4
Lossy(0) or Lossless(1) realizaton req'd:?0
Topology 1,2,3,4,5 OR 6
Branch 1?1
Value :R(Even)/G(Odd)?1
Branch 2?3
Value :L?1
Branch 3?4
Values:L,1/Ql,Rl,C,1/Qc,Gc,Gl?1,0,1,1,0,0,0
Branch 4?5
Values:L,Rs?1,1
```

```
Optimization strategy required
*************************
Least square method:1   Fletcher-Powell method:2?
1
Read i/p data for E04GAF l-s optm.method
Method 1,2,3=?1
Iter-print 0,1=?0
Maxcal=?1E12
Ifail=? 0
```

```
Final sum of squares is .8150598E-14

At the prototype element values
    3.000000
    0.694954
    0.101510
    1.221068
    0.092153
    0.698675

Residuals are
    0.000000
    0.000000
    0.000000
```

This has error number 0
End of E04GAF optimization process

No.of VTF analyses were : 132

Num coeffs (Lowest degree 1st) :

1.000000E+00s^0
1.125250E-01s^1
1.239510E-01s^2

Den coeffs (Lowest degree 1st) :

4.000000E+00s^0
5.852056E+00s^1
4.365148E+00s^2
2.296884E+00s^3

Coefficient sensitivity analysis:

```
Elem:   1:Num: .000000E+00 .000000E+00 .000000E+00
          Den: .750000E+00 .772115E+00 .776163E+00 .100000E+01
Elem:   2:Num: .000000E+00 .000000E+00 .000000E+00
          Den: .000000E+00 .356262E+00 .636945E+00 .886888E+00
Elem:   3:Num: .000000E+00 .000000E+00 .100000E+01
          Den: .000000E+00 .000000E+00 .113582E+00 .225621E+00
Elem:   4:Num: .000000E+00 .100000E+01 .100000E+01
          Den: .000000E+00 .285569E+00 .100000E+01 .100000E+01
Elem:   5:Num: .000000E+00 .100000E+01 .000000E+00
          Den: .000000E+00 .769131E-01 .107775E+00 .000000E+00
Elem:   6:Num: .000000E+00 .000000E+00 .000000E+00
          Den: .000000E+00 .358169E+00 .249473E+00 .887491E+00
Elem:   7:Num: .000000E+00 .000000E+00 .000000E+00
          Den: .750000E+00 .266341E+00 .668388E+00 .000000E+00
```

```
Transfer function sensitivites analyses:
No.of frequency ranges?1

Start,stop freqs.(Rad/sec)?0,5

No.points do you want:(Max=500)?500

Ampl T.F sens: maximum (Range 0.0 to  5.0 Rad/sec)      AT W=1
       Elem. 1:    -0.9904 at  5.0000 Rad/s          -0.6295
       Elem. 2:    -1.0321 at  1.8100 Rad/s          -0.0928
       Elem. 3:     1.6232 at  3.4500 Rad/s          -0.0091
       Elem. 4:    -2.2316 at  2.2900 Rad/s          -0.0736
       Elem. 5:     0.9681 at  2.8400 Rad/s          -0.1225
       Elem. 6:    -0.8494 at  5.0000 Rad/s          -0.0472
       Elem. 7:    -0.7500 at  0.0000 Rad/s          -0.4315
Max. Ampl.T.F. sensitivity:  -2.2316 at  2.2900  Rad/sec' - Elem.  4
Max.Ampl.sens. at W=1 Rad/sec:   -0.6295 - Elem  1
```

FDNR sensitivity required ?1

```
Branch  1:
Elmt.  1:C:.3000000E+01
Branch  2:
Elmt.  2:R:.6949541E+00
Branch  3:
Elmt.  3:R:.1015103E+00
FDNR ellipt, 4, 5 and  6 : C1(&C3) and R2:.2170306E+02.2592376E-02
Branch  4:
Elmt.  7:R:.6986751E+00
Elmt.  8:C:.1000000E+01

Rmax is :        6.986751E-01     Rmin is :        2.592376E-03
Cmax is :        2.170306E+01     Cmin is :        1.000000E+00
R-spread is :    2.695115E+02     C-spread is :    2.170306E+01
```